AP Calculus AB Summer Work

About This Packet
(…and this class)

Welcome!

This packet includes a sampling of problems that students *entering* Calculus should be able to answer.

In Calculus, it's rarely the calculus that'll get you; it's the algebra. Students entering Calculus absolutely *must* have a strong foundation in algebra. Most questions in this packet were included because they concern skills and concepts that will be used extensively in Calculus. Others have been included not so much because they are skills that are used frequently, but because being able to answer them indicates a strong grasp of important mathematical concepts and—more importantly—the ability to problem-solve.

An answer key to this packet will be posted to the school website on or about August 1st. This packet will <u>not</u> be collected, but you must bring it to class the first day anyway. (If you're the sort of student who doesn't do homework unless forced to, Calculus might not be the best place for you…) It is extremely important for all students to review the concepts contained in this packet and to be prepared for a <u>test of prerequisite skills will take place on or before our third class.</u> Students whose scores show they were not prepared for the assessment probably either
a) don't have the mathematical prerequisite skills necessary for success in Calculus, or
b) don't have the work ethic necessary for success in Calculus. **Either way, you may be advised to leave the course.**

I will expect you to approach problems with the mathematical *toolkit* needed to do the calculations and the mathematical *understanding* needed to make sense of unusual problems. This is <u>not</u> a class where every problem you see on tests and quizzes is identical to problems you've done dozens of times in class. This is because the AP test itself (and, truly, all "real" mathematics) requires you to take what you know and apply it, rather than to simply regurgitate a rote process.

Now that I've said all that, I encourage you to <u>take a deep breath</u> and start working. If you have the basics down and you put in the work needed, you'll see how amazing Calculus is! AP Calculus is challenging, demanding, rewarding, and—to put it simply—totally awesome.

A: Super-Basic Algebra Skills

A1. **True or false.** If false, change what is underlined to make the statement true.

a. $\left(x^3\right)^4 = \underline{x^{12}}$ T F

b. $x^{\frac{1}{2}}x^3 = \underline{x^{\frac{3}{2}}}$ T F

c. $(x+3)^2 = \underline{x^2+9}$ T F

d. $\dfrac{x^2-1}{x-1} = \underline{x}$ T F

e. $(4x+12)^2 = \underline{16}(x+3)^2$ T F

f. $\underline{3} + 2\sqrt{x-3} = 5\sqrt{x-3}$ T F

g. If $(x+3)(x-10) = \underline{\ 2\ }$, then $x+3 = \underline{\ 2\ }$ or $x-10 = \underline{\ 2\ }$. T F

A2. More basic algebra.

a. If 6 is a zero of f, then ____ is a solution of $f(2x) = 0$.

b. Lucy has the equation $2(4x+6)^2 - 8 = 16$. She multiplies both sides by ½. If she does this correctly, what is the resulting equation?

c. Simplify $\dfrac{2 \pm 4\sqrt{10}}{2}$

d. Rationalize the denominator of $\dfrac{12}{3+\sqrt{x-1}}$

e. If $f(x) = 3x^2 + x + 5$, then $f(x+h) - f(x) =$ (Give your answer in simplest form.)

f. A cone's volume is given by $V = \frac{1}{3}\pi r^2 h$. If $r = 3h$, write V in terms of h.

T: Trigonometry

You should be able to answer these quickly, *without* referring to a unit circle.

T1. Find the value of each expression, in exact form.

a. $\sin \frac{2\pi}{3}$

b. $\cos \frac{11\pi}{6}$

c. $\tan \frac{3\pi}{4}$

d. $\sec \frac{5\pi}{3}$

e. $\csc \frac{7\pi}{4}$

f. $\cot \frac{5\pi}{6}$

T2. Find the value(s) of x in $[0, 2\pi)$ which solve each equation.

a. $\sin x = \frac{\sqrt{3}}{2}$

b. $\cos x = -1$

c. $\tan x = \sqrt{3}$

d. $\sec x = -2$

e. $\csc x$ is undefined

f. $\cot x = 1$

T3. Solve the equation. Find the value(s) of x in $[0, 2\pi)$, if any.

a. $\sin 3x = 1$

b. $2\sqrt{3}\cos(\pi x) = 3$

c. $\tan 2x = 0$

d. $4\sec x + 1 = 9$

e. $\csc(4x + 3) = 0$

f. $3\cot 6x + \sqrt{3} = 0$

T4. Solve by factoring. Find the value(s) of x in $[0, 2\pi)$, if any.

a. $4\sin^2 x + 4\sin x + 1 = 0$

b. $\cos^2 x - \cos x = 0$

c. $\sin x \cos x - \sin^2 x = 0$

d. $x\tan x + 3\tan x = x + 3$

F: Higher-Level Factoring

F1. Solve by factoring.

a. $x^3 + 5x^2 - x - 5 = 0$

b. $4x^4 + 36 = 40x^2$

c. $(x^3 - 6)^2 + 3(x^3 - 6) - 10 = 0$

d. $x^5 + 8 = x^3 + 8x^2$

F2. Solve by factoring. You should be able to solve each of these *without* multiplying the whole thing out. (In fact, for goodness' sake, please *don't* multiply it all out!)

a. $(x + 2)^2 (x + 6)^3 + (x + 2)(x + 6)^4 = 0$

b. $(2x - 3)^3 (x^2 - 9)^2 + (2x - 3)^5 (x^2 - 9) = 0$

c. $(3x + 11)^5 (x + 5)^2 (2x - 1)^3 + (3x + 11)^4 (x + 5)^4 (2x - 1)^3 = 0$

d. $6x^2 - 5x - 4 = (2x + 1)^2 (3x - 4)^2$

F3. Solve. Each question *can* be solved by factoring, but there are other methods, too.

a. $a(3a + 2)^{1/2} + 2(3a + 2)^{3/2} = 0$

b. $\sqrt{2x^2 + x - 6} + \sqrt{2x - 3} = 0$

c. $2\sqrt{x + 3} = x + 3$

L: Logarithms and Exponential Functions

L1. Expand as much as possible.

a. $\ln x^2y^3$

b. $\ln \frac{x+3}{4y}$

c. $\ln 3\sqrt{x}$

d. $\ln 4xy$

L2. Condense into the logarithm of a single expression.

a. $4\ln x + 5\ln y$

b. $\frac{2}{3}\ln a + 5\ln 2$

c. $\ln x - \ln 2$

d. $\frac{\ln x}{\ln 2}$
(contrast with part **c**)

L3. Solve. Give your answer in exact form.

a. $\ln (x+3) = 2$

b. $\ln x + \ln 4 = 1$

c. $\ln x + \ln (x+2) = \ln 3$

d. $\ln (x+1) - \ln (2x-3) = \ln 2$

L4. Solve. Give your answer in exact form.

a. $e^{4x+5} = 1$

b. $2^x = 8^{4x-1}$

c. $100e^{x\ln 4} = 50$

R: Rational Expressions and Equations

R1.

	Function	Domain	Horiz. Asym., if any	Vert. Asym.(s), if any
a.	$f(x)=\dfrac{4x^2+7x-15}{8x^2-14x+5}$			
b.	$f(x)=\dfrac{3(4+x)^2-48}{x}$			
c.	$f(x)=\dfrac{6x+4}{\sqrt{3x^2-10x-8}}$		skip	

R3. Find the x-coordinates where the function's output is zero and where it is undefined.

a. For what real value(s) of x, if any, is the output of the function $f(x)=\dfrac{x^2+4}{e^{6x}-1}$

…equal to zero? …undefined?

b. For what real value(s) of x, if any, is the output of $g(x)=\dfrac{\cos^2(\pi x)}{\sin x+2}$…

…equal to zero? …undefined?

R4. Simplify completely.

a. $\dfrac{2}{\sqrt{x^2+4}}-\dfrac{x^2+4}{3}$ (Don't worry about rationalizing)

b. $\dfrac{3}{\left(\frac{4}{x}\right)^2+1}$ (Your final answer should have just one numerator and one denominator)

c. $\dfrac{5}{x^2+3x+2}-\dfrac{2x}{x^2+2x+1}$

d. $\dfrac{3}{(x+2)^{1/2}}+\dfrac{x}{(x+2)^{5/2}}$ (Don't worry about rationalizing)