A. Simplifying Polynomial Expressions

I. Combining Like Terms

- You can add or subtract terms that are considered "like", or terms that have the same variable(s) with the same exponent(s).

    Ex. 1:   5x - 7y + 10x + 3y

    5x - 7y + 10x + 3y

    15x - 4y

    Ex. 2: $-8h^2 + 10h^3 - 12h^2 - 15h^3$

    $-8h^2 + 10h^3 - 12h^2 - 15h^3$

    $-20h^2 - 5h^3$

II. Applying the Distributive Property

- Every term inside the parentheses is multiplied by the term outside of the parentheses.

    $Ex. 1: 3(9x - 4)$

    $3 \cdot 9x - 3 \cdot 4$

    $27x - 12$

    $Ex. 2: 4x^2(5x^3 + 6x)$

    $4x^2 \cdot 5x^3 + 4x^2 \cdot 6x$

    $20x^5 + 24x^3$

III. Combining Like Terms AND the Distributive Property (Problems with a Mix!)

- Sometimes problems will require you to distribute AND combine like terms!!

    $Ex. 1: 3(4x - 2) + 13x$

    $3 \cdot 4x - 3 \cdot 2 + 13x$

    $12x - 6 + 13x$

    $25x - 6$

    $Ex. 2: 3(12x - 5) - 9(-7 + 10x)$

    $3 \cdot 12x - 3 \cdot 5 - 9(-7) - 9(10x)$

    $36x - 15 + 63 - 90x$

    $-54x + 48$

PRACTICE SET 1

Simplify.

1. $8x - 9y + 16x + 12y$

2. $14y + 22 - 15y^2 + 23y$

3. $5n - (3 - 4n)$

4. $-2(11b - 3)$

5. $10q(16x + 11)$

6. $-(5x - 6)$

7. $3(18z - 4w) + 2(10z - 6w)$

8. $(8c + 3) + 12(4c - 10)$

9. $9(6x - 2) - 3(9x^2 - 3)$

10. $-(y - x) + 6(5x + 7)$

B. Solving Equations

I. Solving Two-Step Equations

A couple of hints: 1. To solve an equation, UNDO the order of operations and work in the reverse order.
    2. REMEMBER! Addition is "undone" by subtraction, and vice versa. Multiplication is "undone" by division, and vice versa.

$Ex.\,1:\;4x-2=30$
$$\underline{+2\quad+2}$$
$$4x=32$$
$$\underline{\div4\quad\div4}$$
$$x=8$$

$Ex.\,2:\;87=-11x+21$
$$\underline{-21\qquad-21}$$
$$66=-11x$$
$$\underline{\div-11\quad\div-11}$$
$$-6=x$$

II. Solving Multi-step Equations With Variables on Both Sides of the Equal Sign

- When solving equations with variables on both sides of the equal sign, be sure to get all terms with variables on one side and all the terms without variables on the other side.

$$Ex.\,3:\;8x+4=4x+28$$
$$\underline{-4\qquad-4}$$
$$8x=4x+24$$
$$\underline{-4x\quad-4x}$$
$$4x=24$$
$$\underline{\div4\quad\div4}$$
$$x=6$$

III. Solving Equations that need to be simplified first

- In some equations, you will need to combine like terms and/or use the distributive property to simplify each side of the equation, and then begin to solve it.

$$Ex.\,4:\;5(4x-7)=8x+45+2x$$
$$20x-35=10x+45$$
$$\underline{-10x\qquad-10x}$$
$$10x-35=45$$
$$\underline{+35\quad+35}$$
$$10x=80$$
$$\underline{\div10\quad\div10}$$
$$x=8$$

PRACTICE SET 2

Solve each equation. You must show all work.

1. $5x - 2 = 33$

2. $140 = 4x + 36$

3. $8(3x - 4) = 196$

4. $45x - 720 + 15x = 60$

5. $132 = 4(12x - 9)$

6. $198 = 154 + 7x - 68$

7. $-131 = -5(3x - 8) + 6x$

8. $-7x - 10 = 18 + 3x$

9. $12x + 8 - 15 = -2(3x - 82)$

10. $-(12x - 6) = 12x + 6$

IV. Solving Literal Equations

- A literal equation is an equation that contains more than one variable.
- You can solve a literal equation for one of the variables by getting that variable by itself (isolating the specified variable).

$Ex. 1: 3xy = 18,$ Solve for x.

$$\frac{3xy}{3y} = \frac{18}{3y}$$

$$x = \frac{6}{y}$$

$Ex. 2: 5a - 10b = 20,$ Solve for a.

$$+10b = +10b$$

$$5a = 20 + 10b$$

$$\frac{5a}{5} = \frac{20}{5} + \frac{10b}{5}$$

$$a = 4 + 2b$$

PRACTICE SET 3

Solve each equation for the specified variable.

1. $Y + V = W$, for V

2. $9wr = 81$, for w

3. $2d - 3f = 9$, for f

4. $dx + t = 10$, for x

5. $P = (g - 9)180$, for g

6. $4x + y - 5h = 10y + u$, for x

C. Rules of Exponents

Multiplication: Recall $(x^m)(x^n) = x^{(m+n)}$ Ex: $(3x^4y^2)(4xy^5) = (3 \cdot 4)(x^4 \cdot x^1)(y^2 \cdot y^5) = 12x^5y^7$

Division: Recall $\dfrac{x^m}{x^n} = x^{(m-n)}$ Ex: $\dfrac{42m^5j^2}{-3m^3j} = \left(\dfrac{42}{-3}\right)\left(\dfrac{m^5}{m^3}\right)\left(\dfrac{j^2}{j^1}\right) = -14m^2j$

Powers: Recall $(x^m)^n = x^{(m \cdot n)}$ Ex: $(-2a^3bc^4)^3 = (-2)^3(a^3)^3(b^1)^3(c^4)^3 = -8a^9b^3c^{12}$

Power of Zero: Recall $x^0 = 1, x \neq 0$ Ex: $5x^0y^4 = (5)(1)(y^4) = 5y^4$

PRACTICE SET 4

Simplify each expression.

1. $(c^5)(c)(c^2)$

2. $\dfrac{m^{15}}{m^3}$

3. $(k^4)^5$

4. d^0

5. $(p^4q^2)(p^7q^5)$

6. $\dfrac{45y^3z^{10}}{5y^3z}$

7. $(-t^7)^3$

8. $3f^3g^0$

9. $(4h^5k^3)(15k^2h^3)$

10. $\dfrac{12a^4b^6}{36ab^2c}$

11. $(3m^2n)^4$

12. $(12x^2y)^0$

13. $(-5a^2b)(2ab^2c)(-3b)$

14. $4x(2x^2y)^0$

15. $(3x^4y)(2y^2)^3$

D. Binomial Multiplication

I. Reviewing the Distributive Property

The distributive property is used when you want to multiply a single term by an expression.

$$Ex\ 1:\ 8(5x^2 - 9x)$$
$$8 \cdot 5x^2 + 8 \cdot (-9x)$$
$$40x^2 - 72x$$

II. Multiplying Binomials – the FOIL method

When multiplying two binomials (an expression with two terms), we use the "FOIL" method. The "FOIL" method uses the distributive property twice!
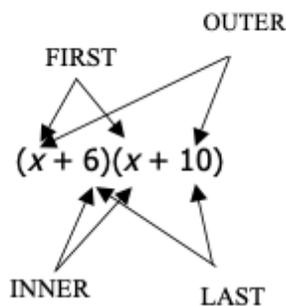
FOIL is the order in which you will multiply your terms.

F_{irst}

O_{uter}

I_{nner}

L_{ast}

Ex. 1: (x + 6)(x + 10)

First	$x \cdot x$ ------> x^2
Outer	$x \cdot 10$ -----> $10x$
Inner	$6 \cdot x$ ------> $6x$
Last	$6 \cdot 10$ -----> 60

$$x^2 + 10x + 6x + 60$$

$$x^2 + 16x + 60$$
(After combining like terms)

Recall: $4^2 = 4 \cdot 4$

$$x^2 = x \cdot x$$

Ex. $(x + 5)^2$

$(x + 5)^2 = (x + 5)(x+5)$ Now you can use the "FOIL" method to get
a simplified expression.

PRACTICE SET 5

Multiply. Write your answer in simplest form.

1. $(x + 10)(x - 9)$ 2. $(x + 7)(x - 12)$

3. $(x - 10)(x - 2)$ 4. $(x - 8)(x + 81)$

5. $(2x - 1)(4x + 3)$ 6. $(-2x + 10)(-9x + 5)$

7. $(-3x - 4)(2x + 4)$ 8. $(x + 10)^2$

9. $(-x + 5)^2$ 10. $(2x - 3)^2$

E. Factoring

I. Using the Greatest Common Factor (GCF) to Factor.

- Always determine whether there is a greatest common factor (GCF) first.

    Ex. 1 $3x^4 - 33x^3 + 90x^2$

    - In this example the GCF is $3x^2$.
    - So when we factor, we have $3x^2(x^2 - 11x + 30)$.
    - Now we need to look at the polynomial remaining in the parentheses. Can this trinomial be factored into two binomials? In order to determine this make a list of all of the factors of 30.

30		30	
1	30	-1	-30
2	15	-2	-15
3	10	-3	-10
5	6	-5	-6

        Since $-5 + -6 = -11$ and $(-5)(-6) = 30$ we should choose -5 and -6 in order to factor the expression.

    - The expression factors into $3x^2(x - 5)(x - 6)$

    Note: Not all expressions will have a GCF. If a trinomial expression does not have a GCF, proceed by trying to factor the trinomial into two binomials.

II. Applying the difference of squares: $a^2 - b^2 = (a - b)(a + b)$

Ex. 2 $4x^3 - 100x$

$4x(x^2 - 25)$

$4x(x - 5)(x + 5)$

Since x^2 and 25 are perfect squares separated by a subtraction sign, you can apply the difference of two squares formula.

PRACTICE SET 6

Factor each expression.

1. $3x^2 + 6x$

2. $4a^2b^2 - 16ab^3 + 8ab^2c$

3. $x^2 - 25$

4. $n^2 + 8n + 15$

5. $g^2 - 9g + 20$

6. $d^2 + 3d - 28$

7. $z^2 - 7z - 30$

8. $m^2 + 18m + 81$

9. $4y^3 - 36y$

10. $5k^2 + 30k - 135$

F. Radicals

To simplify a radical, we need to find the greatest perfect square factor of the number under the radical sign (the radicand) and then take the square root of that number.

$Ex. 1: \sqrt{72}$
$\sqrt{36} \cdot \sqrt{2}$
$6\sqrt{2}$

$Ex. 2: 4\sqrt{90}$
$4 \cdot \sqrt{9} \cdot \sqrt{10}$
$4 \cdot 3 \cdot \sqrt{10}$
$12\sqrt{10}$

$Ex. 3: \sqrt{48}$
$\sqrt{16}\sqrt{3}$
$4\sqrt{3}$

OR

$Ex. 3: \sqrt{48}$
$\sqrt{4}\sqrt{12}$
$2\sqrt{12}$ ← This is not simplified completely because 12 is divisible by 4 (another perfect square)
$2\sqrt{4}\sqrt{3}$
$2 \cdot 2 \cdot \sqrt{3}$
$4\sqrt{3}$

PRACTICE SET 7

Simplify each radical.

1. $\sqrt{121}$

2. $\sqrt{90}$

3. $\sqrt{175}$

4. $\sqrt{288}$

5. $\sqrt{486}$

6. $2\sqrt{16}$

7. $6\sqrt{500}$

8. $3\sqrt{147}$

9. $8\sqrt{475}$

10. $\sqrt{\dfrac{125}{9}}$

G. Graphing Lines

I. Finding the Slope of the Line that Contains each Pair of Points.

Given two points with coordinates (x_1, y_1) and (x_2, y_2), the formula for the slope, m, of the line containing the points is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

Ex. (2, 5) and (4, 1)

$$m = \frac{1-5}{4-2} = \frac{-4}{2} = -2$$

The slope is -2.

Ex. (-3, 2) and (2, 3)

$$m = \frac{3-2}{2-(-3)} = \frac{1}{5}$$

The slope is $\dfrac{1}{5}$

PRACTICE SET 8

1. (-1, 4) and (1, -2)

2. (3, 5) and (-3, 1)

3. (1, -3) and (-1, -2)

4. (2, -4) and (6, -4)

5. (2, 1) and (-2, -3)

6. (5, -2) and (5, 7)

II. Using the Slope – Intercept Form of the Equation of a Line.

The slope-intercept form for the equation of a line with slope m and y-intercept b is $y = mx + b$.

Ex. $y = 3x - 1$

Slope: 3 y-intercept: -1
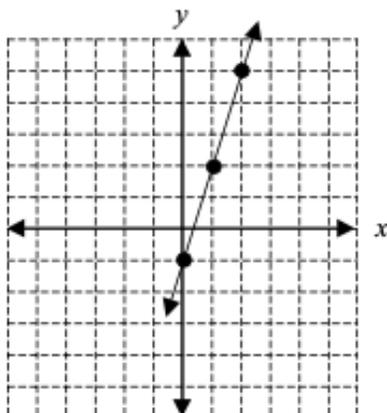
Place a point on the y-axis at -1.
Slope is 3 or 3/1, so travel up 3 on
the y-axis and over 1 to the right.

Ex. $y = -\dfrac{3}{4}x + 2$
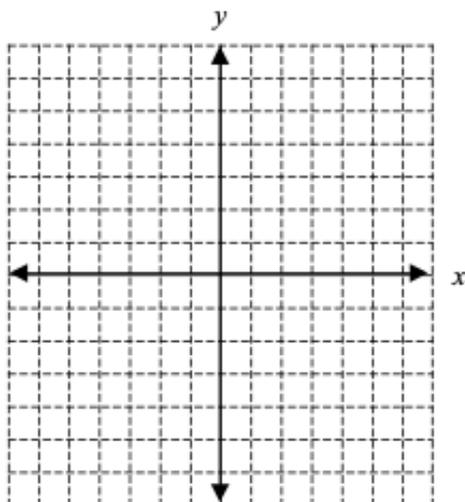
Slope: $-\dfrac{3}{4}$ y-intercept: 2
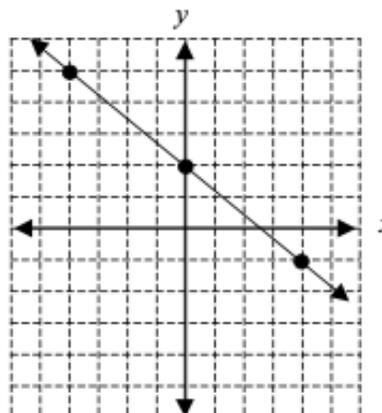
Place a point on the y-axis at 2.
Slope is -3/4 so travel down 3 on the
y-axis and over 4 to the right. Or travel
up 3 on the y-axis and over 4 to the left.

PRACTICE SET 9

1. $y = 2x + 5$

Slope: _____ y-intercept: _____

2. $y = \dfrac{1}{2}x - 3$

Slope: _____ y-intercept: _____

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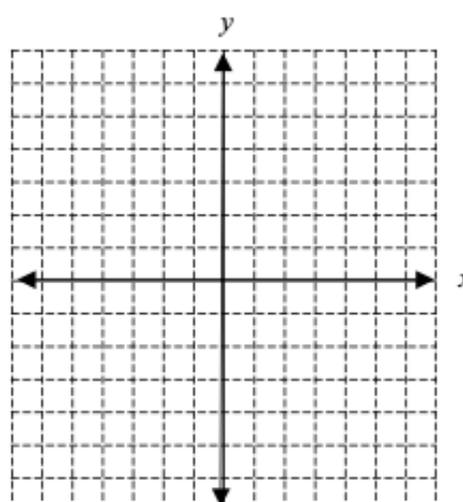
3. $y = -\dfrac{2}{5}x + 4$

Slope: _____

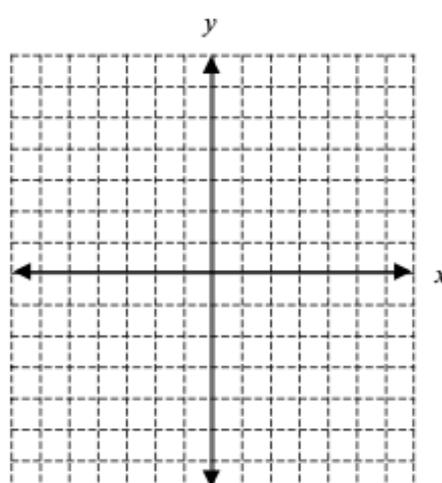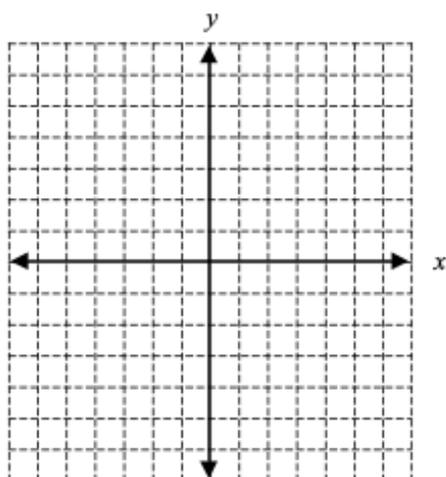y-intercept: _____
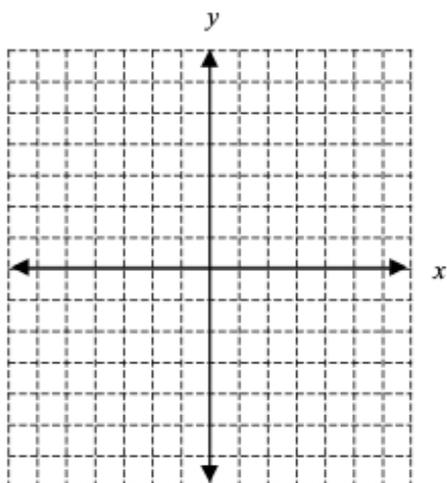
4. $y = -3x$

Slope: _____

y-intercept _____

5. $y = -x + 2$

Slope: _____

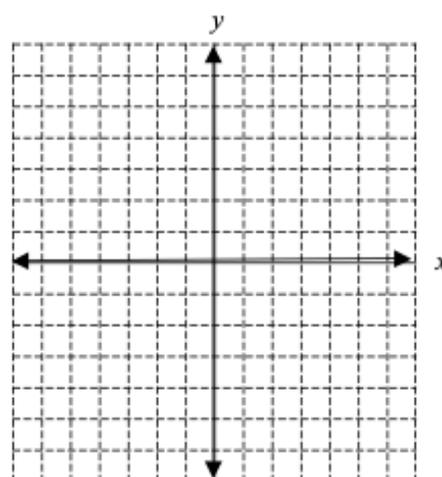y-intercept: _____

6. $y = x$

Slope: _____

y-intercept _____

III. Using Standard Form to Graph a Line.

An equation in standard form can be graphed using several different methods. Two methods are explained below.

    a. Re-write the equation in $y = mx + b$ form, identify the y-intercept and slope, then graph as in Part II above.

    b. Solve for the x- and y- intercepts. To find the x-intercept, let $y = 0$ and solve for x. To find the y-intercept, let $x = 0$ and solve for y. Then plot these points on the appropriate axes and connect them with a line.

Ex. $2x - 3y = 10$

a. Solve for y.   OR   b. Find the intercepts:

$$-3y = -2x + 10$$

$$y = \frac{-2x + 10}{-3}$$

$$y = \frac{2}{3}x - \frac{10}{3}$$

let $y = 0$:

$$2x - 3(0) = 10$$
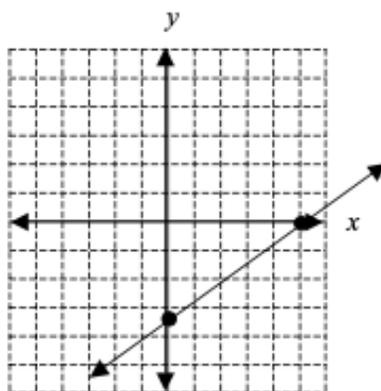
$$2x = 10$$

$$x = 5$$

So x-intercept is (5, 0)

let $x = 0$:

$$2(0) - 3y = 10$$

$$-3y = 10$$

$$y = -\frac{10}{3}$$
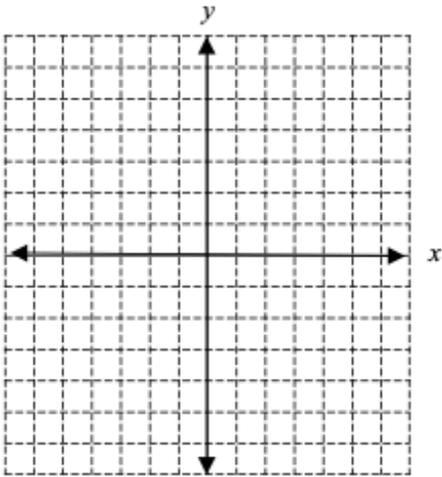
So y-intercept is $\left(0, -\frac{10}{3}\right)$

On the x-axis place a point at 5.

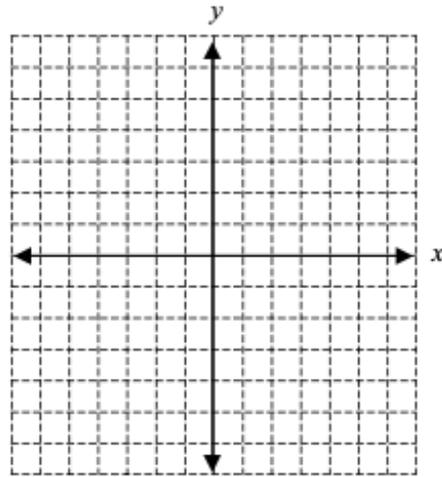On the y-axis place a point at $-\frac{10}{3} = -3\frac{1}{3}$

Connect the points with the line.

PRACTICE SET 10

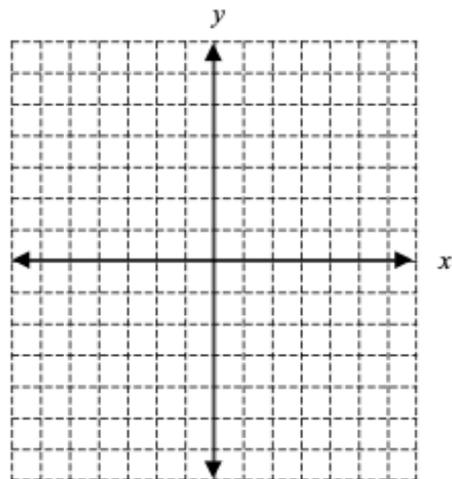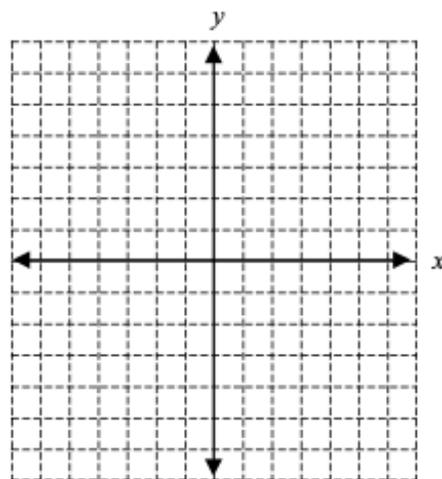1. $3x + y = 3$

2. $5x + 2y = 10$

3. $y = 4$

4. $4x - 3y = 9$

Proportions

Setting up:
1. Create two equal ratios.
2. **Label** each numerator and denominator so that they **match**.

Solving:
1. **Cross multiply** by multiplying the two diagonals and set them equal to one another.
2. **Solve.**

$$\frac{3}{5} = \frac{x}{10} \qquad 3 \bullet 10 = 5 \bullet x$$

$$30 = 5x$$
$$6 = x$$

Solve. Show All Work!

1. $\dfrac{x}{2\frac{1}{3}} = \dfrac{8}{3}$	2. $\dfrac{6}{2} = \dfrac{4}{x}$
3. $\dfrac{5.1}{1.7} = \dfrac{7.5}{x}$	4. $\dfrac{6.4}{0.8} = \dfrac{8.1}{x}$
5. Given the scale 2 cm = 3 m, how long is the scale drawing of a basketball rim that is 16 m. tall?	6. To tie dye a shirt orange, you need 2 parts red to 5 parts yellow. How much yellow do you need if you have 13 parts red?
7. Find the unit price of a case (12 cans) of soda for $2.25.	8. A rectangle is 11.4 in tall and 5.4 in wide. If it is reduced to a height of 5.7 in then how wide will it be?

Word Problems: Solve by forming an algebraic equation to find the unknow parts.

1. The length of a rectangle is 6 feet greater than the width. The perimeter is 40 ft. Find the length and the width.

2. The length of a rectangle is twice the width. The perimeter is 42 cm. Find length and the width.

3. The length of a rectangle is 2 yards less than 3 times the width. The perimeter is 68 yards. Find the length and the width.

4. The first side of a triangle is 2 feet longer than the second. The third side is 5 feet shorter than twice the second. The perimeter is 49 feet. Find the length of each side.

5. One angle of a triangle is twice as large as another. The third angle contains 5° more than the larger of these. Find each angle.

6. One angle of a triangle is 20° more than another angle. The third angle is 5° less than the smaller of these. Find each angle.

7. The perimeter of a rectangle is 86 inches. If the length is 3 inches longer than the width, find the lengths of each side.

8. The second of two numbers is 4 times the first. Their sum is 50. Find the numbers.